Stressed to the Breaking Point

(Multiple-Variable Functions)

Objective

Students will explore the relationship between the amount of weight that can be supported by a spaghetti bridge, the thickness of the bridge, and the length of the bridge to determine the algebraic equation that best represents that pattern modeled by the three variables.

Overview of the Lesson

How does the amount of weight that can be supported by a spaghetti bridge relate to the width (number of spaghetti strands) and the length of the bridge? Students gather data comparing the amount of weight that can be supported, the number of strands of spaghetti used, and the length of the bridge. They explore the relationship between the amount of weight that can be supported and the number of strands used (the width of the bridge) and discover that this relationship is linear. Students make a scatterplot and determine the equation for this relationship. Students also explore the relationship between the amount of weight that can be supported and the length of the bridge. Again, they make a scatterplot and determine that as the length of the bridge increases, the amount of weight that can be supported decreases. Students explore in detail both relationships—the linear and the inverse models—and they determine how the three variables—weight, length of the bridge, and width or number of strands of spaghetti—are related and expressed in one equation.

Materials

- graphing calculator overhead unit
- overhead projector
- class data chart on newsprint or blackboard

For each group of four:

- cup with paper clips for hanging over the spaghetti bridges
- several rolls of pennies
- a handful of Vermicelli (thin spaghetti)
- a balance scale (optional)
- *Stressed to the Breaking Point* activity sheets

Procedure

1. **Introduction**: Distribute the *Stressed to the Breaking Point* activity sheets and have students read and discuss the karate chop questions from the **Think About This Situation.** This section is designed to stimulate students to think about a situation where there are more than two variables to consider. Some students will probably have a fairly good intuitive sense about which board will require the greatest energy to break, but they probably have not thought about the fact that the amount of energy required is related directly to the thickness of the board and inversely to the length. In real-world problem situations there are often many variables operating at the same time.

2. **Assigning Groups and Distributing Materials**: Have the students work in groups of four. It is helpful to organize the class so that each group collects data for a given span or distance. Spans of 2, 2.5, 3, 3.5, 4, 4.5 and 5 inches are recommended as good lengths to use because using very wide spans is difficult since spaghetti is thin. Using a constant span allows the group to set two desks a given distance apart and simply vary the number of strands of spaghetti that are used. Vermicelli is highly recommended. If you use thicker spaghetti, it is very difficult to get the very short, thick bridges to break. Also, you may wish to have students put newspaper down on the floor to collect the broken spaghetti. Your custodian will appreciate this! Groups may use pennies to represent the amount of weight that can be supported, or they can use a balance scale to actually measure this amount. The containers for the pennies are made by attaching three paper clips to the top of a cup spaced equidistant apart and then joining these together with a fourth paper clip which will be able to slip over the spaghetti. Students could make these, but it is suggested that this be done ahead of time. The materials needed for each group can be organized in tote trays in advance to make distributing them fast and easy.

3. **Collecting Data**: Once the students have the materials they need, and their span length assignment, they should collect their data, varying the number of strands while keeping the span length constant. Caution the students not to hold the ends of the spaghetti. These ends will bow up as more weight is added, but the spaghetti will eventually break. Students may notice that the spaghetti varies slightly in thickness. Encourage students to avoid pieces of spaghetti that seem much thicker or thinner than most others. As the students gather their data, have them enter it on their activity sheet and on

the class data chart which could be recorded on newsprint, or simply put on the blackboard.

4. **Looking for Patterns:** Students should work in groups to discover the patterns represented in the table. In order to divide the work involved in the data analysis, it is helpful to assign each group a particular row of data for problem number 1, and a particular column of data for problem number 2. Once all groups have had time to use their calculators to make their scatterplots and determine their mathematical models, the class should discuss the findings of each group. The important point is that the pattern that describes how the breaking weight changes as the number of strands increases is best represented by a linear model. In exploring the pattern that describes how the breaking weight changes as the span increases, students should find that the breaking weight decreases as the span increases. This relationship is best represented by a power model, but some students might use an exponential model here. It is important to discuss as an entire class the mathematical patterns found by each group. Ask the students to consider characteristics of power models and exponential models that might give them ideas about which would be the better model. Question the students to illicit important ideas, especially if some students seem to be having trouble.

5. **Putting It All Together**: Students explore which single equation best expresses the relation between the breaking weight, bridge length, and bridge thickness. For some groups, this will involve trial and error testing of the proposed models. Reasoning may help them eliminate some possibilities without actually filling in a chart of values. Students should understand that they are looking for a model where W(weight) increases as T(thickness) increases, and W decreases as L(length) increases. Hopefully, the students will be able to see that $W = 10T + L$ and $W = 10T \times L$ are models where W will increase as L increases, and therefore they are not options. Students might want to actually test the two remaining models by completing the chart to determine the patterns for both $W = 10T - L$ and $W = 10T/L$. By studying the completed charts students should notice that when $W = 10T - L$, the patterns are all linear because the change is constant. However, with the second model $W = 10T/L$, as L increases W decreases, but not at a constant rate. This second model is more like the data pattern from the investigation. The class should discuss the findings and the reasoning of the group work.

Assessment

The continuous process of evaluation is part of good teaching. In this lesson it is important to monitor the groups as they collect data, and work to analyze that data. It is also important to listen to the students as they put forth their ideas in the class discussions. Knowing where they are with their thinking is important for what the teacher does in any given lesson, but it is also important for planning future lessons.

Assessment for the purpose of evaluating students was not directly evident in this lesson, but it could be included. For example, each student could be asked to write the answer to the questions in the investigation and turn them in at the end of class. Students might also be asked to explain the connection between the karate experiment and the bridge investigation.

Extensions & Adaptations

- In this lesson, students determine that $W = k(L/T)$ is the general pattern that represents the relationship between the weight the bridge can support, the length of the bridge, and the thickness of the bridge. Have the students determine the value for k based on the data collected by the class.

- Ask students to explain what their experiment results tell them about the relationships of the variables in the following formulas.

$$DENSITY = \frac{MASS}{VOLUME}$$

$$PRESSURE = \frac{FORCE}{AREA}$$

$$RATE = \frac{DISTANCE}{TIME}$$

- The lesson can be modified. For example. students could gather data and plot the scatterplots for the weight and length data, and the weight and thickness data. In the first case they could see that the relationship is nonlinear. In the second case they could recognize a linear pattern and determine the model.

Mathematically Speaking

In this particular experiment, students are not likely to collect data that fits a model perfectly. For example, the relationship between the weight that can be supported by the bridge and the width of the bridge should be a linear function, but some students may find that an exponential model fits their data better. It is important for students to examine all of the data and have the entire class decide on the general trends that are represented.

The relationship between the weight that can be supported by the bridge and the length of the spaghetti bridge usually gives a scatterplot that looks close to both an exponential and an inverse power model. Help the students think about the characteristics of those two models in deciding which model to use. Even if the correlation coefficient for the exponential model is higher, the inverse model makes more sense if you think about what happens as the length of the bridge approaches zero. For an exponential model, if the length of the bridge is zero there will be a certain amount of weight that the bridge will support. For the model $W = a(b)^L$, that value would be *a.* The students should agree that this is not the case. They should also agree that if the length of the bridge is 0, then no bridge is defined, so there would be no amount of weight defined either, and thus the inverse power model seems to make more sense considering the physical situation.

Identifying these general trends is an important aspect of students' growth in the understanding of functions. Many times when students see a curve they automatically think that the function is a parabola. This is probably because for many students, the first function they study that is not linear is a quadratic. As students grow in their understanding of functions, they should have a solid notion about the patterns represented by the symbolic rules, graphs, and tables for linear, power, quadratic, exponential, trigonometric, and logarithmic functions.

Tips From Ellen

Assessment of Group Work

In many lessons, when it comes to assessment of learning during class time, the magic words are "teacher observation." This is particularly true when students are engaged in group work. That raises the questions:

What is being observed?
What are the desired behaviors?

A very simple concept that can be used to make "teacher observation" a very practical and powerful assessment tool is called "clipboard cruising."

As with all assessment, the first step should be to identify the target. This might be done in a generic sense, just as a rubric can be developed generically. Behaviors might be content specific:

- Uses mathematical terms with precision
- Reasons logically
- Seeks accuracy

Behaviors might also revolve around collaborative problem solving skills:

- Offers ideas
- Listens to and builds on the ideas of others
- Encourages the group

The next step is to develop an understanding with students of what each skill might look like and sound like. For instance, *seeks accuracy* might look like calculating an answer a second time to double check the results, or might sound like, "Lets move the graduated cylinder to a level surface before we do our measurements." *Listens to and builds on the ideas of others* might look like writing down a group member's suggestion accurately, and might sound like, "I like Elaine's idea about creating a table, and I would suggest that we write it on the chalkboard so that we can all read it at the same time." At the same time, make it clear how results will be used. They may be used as part of a class participation grade, as a grade in mathematics communication skills or mathematical dispositions, or simply for group processing at the end of a class.

The third step is to select a skill that is essential to successful completion of a task and in which students may need practice, and to announce to students that as you circulate around the room, you will be noting individual and group use of the skill. You will need a clipboard, a roster, and space to make check marks and/or to note behaviors as they are observed.

The final step is to feed back results. This may be done through interviews with students either one-on-one or small group interviews. It may also be done through written comments.

Notice that the focus is on presence of positive behaviors, rather than absence of negative ones. Teachers practicing “clipboard cruising” are usually amazed and pleased with the results. Initial reactions among students when you approach a group, clipboard in hand, are sometimes superficial or frivolous, but behaviors which are reinforced and practiced become internalized and authentic quickly.

Resources

Coxford, Art, James Fey, Christian Hirsch, and Harold Schoen, *Multiple-Variable Models*, Core-Plus Mathematics Project. Kalamazoo, MI, Western Michigan University, 1996.

Internet location: **gopher://ericir.syr.edu:70/00/Lesson/Subject/Science/cecsi.148**
This site allows students to investigate the relationship between distance, time, speed, and acceleration. It leads the student through constructing various graphs using different variables, and then has students relate the graphs.

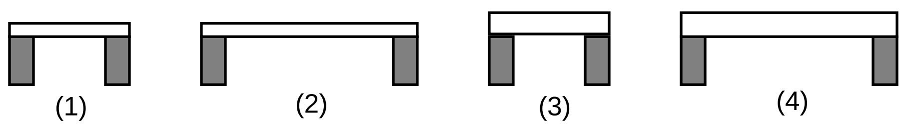
Internet location: **http:///~sboone/Lessons/Titles/indy500.html**
This site allows students to explore the times from the Indy 500 race as they learn data analysis.

Stressed to the Breaking Point

Karate is a very impressive form of the martial arts. Nearly everyone has seen live or video exhibitions of highly trained men and women breaking bricks and boards with chops from their hands or feet or even their heads. Many of us have even tried the karate chops and discovered that they can hurt without proper technique and training.

Think About This Situation

Karate chops break bricks and boards by applying carefully aimed bursts of energy. Different targets require different amounts of energy. Think about the four target boards pictured here:



(a) Which board do you think would require the greatest energy to break?

(b) The board targets differ in length and thickness. How would you expect those two variables to affect required breaking energy?

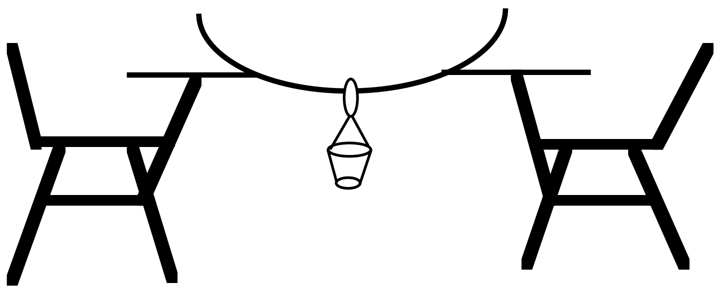
(c) Breaking energy E depends on board length L and thickness T. What sort of equation would you expect to express E as a function of L and T ?

(d) What other variables would you consider in judging the energy required to break a board? How would you expect those variables to be related to each other and to E, L, and T ?

The key factor in those amazing karate exhibitions is actually the speed of the attacking fist or foot. In other situations, strength of a structure like a bridge or beam or suspension cable is measured by loading weight gradually to the breaking point. The breaking weight of any structure certainly depends on the material used. But there are some common patterns relating weight, length, and thickness in every case. You can discover the shape of those relations with a simple experiment!

Experiment: Stressed to the Breaking Point

Get several long pieces of dry pasta (spaghetti, linguine, and vermicelli work well), some paper clips, a paper cup, and a bunch of pennies or fishing weights. With two desktops or tables as supports and pasta as “bridges”, use paper clips to hang the paper cup from the pasta and add weight until the it breaks.



Collecting Data: Find the breaking weight for pieces of pasta that span gaps of different lengths – from 2 - 6 inches. Then try thicker “bridges” by using 2, 3, 4, and 5 strands of pasta. Record the (length, strands, weight) data in a table like this.

Data Table

		Number of Strands				
		1	2	3	4	5
Length in inches	2					
	2.5					
	3					
	3.5					
	4					
	4.5					
	5					

Looking for Patterns: Study the breaking weight data from your experiment, looking for patterns that relate the three variables. Then answer the following questions about algebraic models of the relation between thickness, length, and breaking weight.

1. How does the breaking weight (W) change as the thickness (T) of the pasta “bridge” increases (in numbers of strands)?
   (a) It might help to focus first on the data for 5 inch bridges and several different thicknesses. Use your graphing calculator to make a scatterplot of the (T, W) data and to find an equation modeling the relation between those variables.

(b) Then have members of your group share the work to make scatterplots and find modeling equations relating T and W for "bridges" of other lengths. Compare the results in each case.

(c) Based on the results from (a) and (b), write up your conclusions about the way breaking weight seems to change as the thickness of the "bridge" increases.

2. What do your experimental data say about the way breaking weight (W) changes if only the length (L) of the pasta "bridge" increases?

(a) It might help to focus first on the data for a pasta "bridge" of one strand. Use your graphing calculator to make a scatterplot of the (L, W) data and to find an equation modeling the relation between those variables.

(b) Then have members of your group share the work to make scatterplots and find modeling equations relating L and W for "bridges" of two, three, or four strands. Compare the results in each case.

(c) Based on the results from (a) and (b), write up your conclusions about the way that breaking weight seems to change as the length of the "bridge" increases.

3. Since breaking weight depends on both bridge length and thickness, it would be helpful to express that joint relation with a single equation. Here are four possibilities that were suggested by students in a Kalamazoo class:

$W = 10T + L$ $\quad$ $W = 10T - L$ $\quad$ $W = 10T \times L$ $\quad$ $W = \frac{10T}{L}$

(a) Which of those equations expresses a relationship of W, L, and T similar to what you found in your pasta experiments? In answering this question it might help to use each equation to complete a table showing sample values of W for different combinations of L and T. (See tables.)

		T				
		1	2	3	4	5
L	2					
	2.5					
	3					
	3.5					
	4					
	4.5					
	5					

(b) What relation of W, L, and T matches your data more precisely?

Stressed to the Breaking Point
Data Table

Number of Strands

Length in inches		1	2	3	4	5
	2					
Length	2.5					
in	3					
inches	3.5					
	4					
	4.5					
	5					

W = 10T + L

L \ T	1	2	3	4	5
2					
2.5					
3					
3.5					
4					
4.5					
5					

W = 10T - L

L \ T	1	2	3	4	5
2					
2.5					
3					
3.5					
4					
4.5					
5					

$W = 10T \times L$

L \ T	1	2	3	4	5
2					
2.5					
3					
3.5					
4					
4.5					
5					

$W = 10T \div L$

L \ T	1	2	3	4	5
2					
2.5					
3					
3.5					
4					
4.5					
5					

Stressed to the Breaking Point
Selected Answers

Looking for Patterns

2. (a) *Responses will vary. In order to divide the work involved in the data analysis, it is helpful to assign each group a particular row of data for question 2 and a particular column of data for problem 3. The group should put the mathematical model that they determine on the board to the right of the row or below the column. This will allow the class to see all of the models which will be helpful in recognizing that the pattern that describes how the breaking weight changes as the number of strands increases is best represented by a linear model. It is also both interesting and important to note that the manner in which breaking weight changes as gap length increases is not linear.*

   (b) *Answers will vary.*

   (c) *Breaking weight increases as the thickness, or number of strands, increases. This increase seems to be linear, regardless of the gap length.*

3. (a) *A power model fits the data pattern for this situation. See the discussion in* ***Mathematically Speaking*** *for more details. Some students may find an exponential model that fits well, but in analyzing the physical situation, they should determine that this is not the best model.*

   (b) *Answers will vary.*

   (c) *The breaking weight decreases as the span length increases. The rate of change is not constant.*

4. (a) *Students should understand that they are looking for a model where W increases as T increases, and W decreases as L increases. Hopefully, the students will be able to see that $W = 10T + L$ and $W = 10T \times L$ are models where W increases as L increases and therefore they are not options. Students might want to actually test the two remaining models by completing the chart to determine the patterns for both $W = 10T - L$ and $W = 10T \div L$. By studying the completed charts students should notice that when $W = 10T - L$, the patterns are all linear because the change is constant. However, with the second model $W = 10T \div L$, as L increases W decreases, but not at a constant rate. This second model is more like the data pattern from the investigation.*

   (b) *When L is held constant, $W = 10T \div L$ is simply a linear equation relating W and T. In this way the data-fitting rule is similar to the linear models students have seen already. On the other hand when T is held constant $W = 10T \div L$ is actually a power model relating W and L.*